Test #2

Solid State Physics

May 22, 2025

Solution

You need to have at least a total of 40 points to pass this test

1 Quiz

Select the single *correct answer from the possibilities. Each correct answer is 3 points, each incorrect one is -1 point, no answer is 0 point, but the total points for one question are never negative.*
Maximum points for this part is 30 points

1. We observe two events E_1 followed by E_2 in reference frame $\mathcal{K}$. The square of the space-time interval between these two events is positive. Is it possible to find any $\mathcal{K}'$ reference frame in which E_1 happened after E_2?

   Solution:
   - ☐ *yes, because E_1 causes E_2*
   - ☐ *yes, because E_1 may not cause E_2*
   - ✓ *no, because E_1 may cause E_2*
   - ☐ *no, because E_1 is caused by E_2*

2. Lattice planes

   Solution:
   - ☐ *are created when a crystal breaks*
   - ☐ *are planes with high electron density*
   - ✓ *are imaginary planes through at least 3 non co-linear lattice points*
   - ☐ *separate layers in the crystal with smaller inter-layer binding energy than binding in the plane*

3. The reciprocal lattice

   Solution:
   - ✓ *is used for Bloch electrons*
   - ☐ *is the same lattice, but viewed from the opposite direction*
   - ☐ *when multiplied with the direct lattice the result is 1*
   - ☐ *of an fcc lattice is a different fcc lattice*

4. The Debye model

   Solution:
   - ☐ *is the classical model of conduction*
   - ✓ *assumes velocities of lattice waves are constant up to a limit*
   - ☐ *assumes each atom vibrates with the same frequency*
   - ☐ *results in an exponential dependance of the specific heat capacity on T*

5. The classical physical model of conductivity

   Solution:
   ☐ *is called the Einstein model*
   ☐ *describes phonon-phonon collisions*
   ☐ *uses electrons moving with the Fermi-velocity*
   ✓ *describes non-interactive electrons moving with the drift velocity*

6. The Fermi energy

   Solution:
   ✓ *is the highest occupied energy level in the valence band at T=0 K in metals*
   ☐ *doesn't depend on the electron density*
   ☐ *is in the middle of the energy gap in metals*
   ☐ *is at the top of the energy gap in semiconductors, independent of T*

7. When a Bloch electron moves under the influence of a constant electric field

   Solution:
   ☐ *it can accelerate indefinitely*
   ☐ *after a time it leaves the Brillouin zone of the actual branch it started moving in.*
   ✓ *its crystal momentum oscillates between* $-\frac{\pi}{2}$ *and* $\frac{\pi}{2}$
   ☐ *its velocity becomes zero after every collision with the ion cores*

8. Both holes and electrons can be used together at the same time to describe conduction

   Solution:
   ☐ *in the same band in metals with no band overlap*
   ☐ *in the same band in metals with band overlap*
   ✓ *in different bands only: holes in valence band, electrons in conduction band*
   ☐ *in different bands only: electrons in valence band, holes in conduction band*

9. In extrinsic semiconductors

   Solution:
   ☐ *just a negligible amount of holes are present at room temperature*
   ☐ *the number of holes and electrons are equal*
   ✓ *the product of the electron and hole concentrations equals with the square of the electron concentration without doping*
   ☐ *the Fermi energy at T=0K is in the middle of the energy gap*

10. The generation current of p-n junctions

    Solution:

- ☐ *is equal to the recombination current when forward bias is used*
- ☐ *is equal to the recombination current when reverse bias is used*
- ✓ *is independent of the voltage on the junction*
- ☐ *has an exponential dependance on the voltage on the junction*

2 Problems

Total points for this part is 70 points

Useful constants: elementary charge: $e = 1.6022 \cdot 10^{-19}\ C$,
electron mass: $m_e = 9.1094 \cdot 10^{-31} kg$,
Avogadro's constant L $6.022 \cdot 10^{23}\ 1/mol$

11. The length of the base vectors of the point lattice of a crystal in a Cartesian coordinate system are 8.00 nm and 6.00 nm and the angle between them is 30^o. Give the 4 corners of a possible Wigner-Seitz cell which contains the point selected as the origin of this lattice, both expressed with the base vectors, and by Cartesian coordinates in a coordinate system in which the longer lattice vector lies on the x-axis!

(Hint: A simple drawing of the point lattice and the Wigner-Seitz cell may help.) **(20 points)**

Solution:

Let's denote the two base vectors with $\mathbf{a}_1$ and $\mathbf{a}_2$! The Wigner-Seitz cell is a primitive cell that contains a single lattice point - which is now the origin - in the center and its sides lie at half way between the lattice points.

Then the sides of the Wigner-Seitz cell are parallel with the base vectors, with corner points that are half way between base vectors. The four corners are:

- *bottom left:* $-\frac{1}{2}\left(\mathbf{a}_1 + \mathbf{a}_2\right)$
- *top left:* $-\frac{1}{2}\left(\mathbf{a}_1 - \mathbf{a}_2\right)$
- *top right:* $+\frac{1}{2}\left(\mathbf{a}_1 + \mathbf{a}_2\right)$
- *bottom right:* $+\frac{1}{2}\left(\mathbf{a}_1 - \mathbf{a}_2\right)$

(10 points)

In a Cartesian coordinate system:

$\mathbf{a}_1 = (8.00, 0)\, nm$ *and*

$$\mathbf{a}_2 = (6.00\, cos\, 30^o,\ 6.00\, sin\, 30^o)\, nm = \left(6 \cdot \frac{\sqrt{3}}{2},\ 6\,\frac{1}{2}\right) = (5.20, 3.00)\, nm$$

In this coordinate system the corner coordinates are

- *bottom left:* $\left(-\frac{1}{2}\left(a_{1,x} + a_{2,x}\right),\ -\frac{1}{2}\left(a_{1,y} + a_{2,y},\right)\right)$
- *top left:* $\left(-\frac{1}{2}\left(a_{1.x} - a_{2,x}\right),\ -\frac{1}{2}\left(a_{1.y} - a_{2,y},\right)\right)$
- *top right:* $\left(+\frac{1}{2}\left(a_{1.x} + a_{2,x}\right),\ +\frac{1}{2}\left(a_{1.y} + a_{2,y},\right)\right)$
- *bottom right:* $\left(+\frac{1}{2}\left(a_{1.x} - a_{2,x}\right),\ +\frac{1}{2}\left(a_{1.y} - a_{2,y},\right)\right)$

and because
$\frac{1}{2}\, a_{1,x} = 4.00\, nm$, $\frac{1}{2}\, a_{2,x} = 2.60\, nm$, $\frac{1}{2}\, a_{1,y} = 0\, nm$, $\frac{1}{2}\, a_{2,y} = 1.50\, nm$,
The coordinates are

- *bottom left:* $(-6.60,\ -3.00)\ nm$
- *top left:* $(-1.40,\ 3.00)\ nm$
- *top right:* $(6.60,\ 3.00)\ nm$
- *bottom right:* $(+1.40,\ -3.0)\ nm$

(10 points)

12. In a non-ideal crystal there are many types of crystallographic defects. One of them is a point defect called a vacancy. When the crystal is in thermal equilibrium with the environment what is the ratio of the number of vacancies to the total number of lattice points at 20^oC, if the vacancy formation energy is $\mathcal{E}_{vac} = 1.2eV$? (**15 points**)

Solution:

$$\frac{N_v}{N} = e^{-\dfrac{\mathcal{E}_v}{k_B T}} = e^{-\dfrac{1.2}{0.0258 \cdot 293}} = \underline{\underline{2.2 \cdot 10^{-21}}}$$

13. The lattice constant of a bcc lattice is 0.25 nm. Calculate the volume density of atoms in the conventional (Bravais) unit cell! (**15 points**)

Solution:

Each of the 8 corner atoms belongs to 8 neighboring cells, plus this cubic cell also contains one atom in the middle; therefore, the unit cell contains

$$N = 8 \cdot \frac{1}{8} + 1 = 2 \ atoms$$

The atom density is

$$\rho_{atom} = \frac{N}{V_{cell}} = \frac{2}{(0.25\,nm)^3} = 128\,\frac{atoms}{nm^3} = \underline{\underline{1.28 \cdot 10^{29}\,\frac{atoms}{m^3}}}$$

14. Some specific metal has a mass density of $19.30\,g/cm^3$, a resistivity of $22.14\,n\Omega\,m$, and an electron mobility of $2.59 \cdot 10^{-3}\,\frac{m^2}{V\,s}$. Each atom has 1 valence electron, which can become mobile (free). What is the weight of one mole of this material? (**20 points**)

Solution:

Key point: in this case the electron density, is equal to the density of atoms!

Notations: mass density ρ_m, weight of one mole in kg's A, resistivity ρ, the mobility μ, Avogadro's number L_A.

Let $n[1/m^3]$ be the electron density! Then

$$A = \frac{L_A \cdot \rho_m}{n}$$

n can be calculated from the resistivity and the mobility

$$\mu = \frac{1}{\rho\,e\,n} \quad \Rightarrow \quad n = \frac{1}{\rho\,e\,\mu}$$

$$\begin{aligned} A &= L_A \cdot \rho_m \cdot \rho\,e\,\mu \\ &= 6.02 10^{23}\,1/mol \cdot 19300 kg/m^3 \cdot 22.14 \cdot 10^{-9}\Omega\,m \cdot 1.6022\,10^{-19}\,C \cdot 2.59 \cdot 10^{-3}\,\frac{m^2}{V\,s} \\ &= 0.1068\,\frac{\cancel{C}\,\cancel{\Omega},\cancel{m}\,kg\,\cancel{m^2}}{\cancel{m^3}\,\cancel{V}\,s\,mol} \\ &= 0.1068\,\frac{kg}{mol}{}^1 \end{aligned}$$

That is 1 mol weighs $\underline{\underline{0.1068\ kg}}$.

(The number of atoms and the number of electrons in a m^3 *is then*

$$\begin{aligned} n &= \frac{L_A \cdot \rho_m}{A} = \\ &= 6.0210^{23}\ \mathit{1/mol} \cdot \frac{19300\ \mathit{kg/m^3}}{0.10678\ \mathit{kg/mol}} \\ &= 5.6397 \cdot 10^{24}\ \mathit{1/m^3}) \end{aligned}$$