Test #1

Quantum Mechanics
May 26, 2025
Solution

1 Quiz

Select the correct answer from the possibilities. Each correct answer is 3 points, each incorrect one is -1 point, no answer is 0 point. The total of the the sum of points is never negative.
Total points for this part is 30 points

1. Is it true that if a black body at 1000 K radiates with a power of 100 Watts, then at 2000 K it will radiate 1600 W?

   Solution:
   □ *No, It will only radiate 200 W because* $P \approx T$
   □ *No, It will only radiate 400 W because* $P \approx T^2$
   □ *No, It will only radiate 800 W because* $P \approx T^3$
   ✓ *Yes, It will radiate 1600 W because* $P \approx T^4$

2. The de Broglie formula states: the wavelength λ of an electron with momentum p is

   Solution:
   ✓ h/p
   □ $p/\hbar$
   □ $2\pi p/\hbar$
   □ $2\pi/p$

3. In a central potential angular momentum

   Solution:
   □ *not quantized, because the potential is spherically symmetric*
   ✓ *quantized, any measured components must be an integer or half-integer multiple of* $\hbar$
   □ *quantized, the length is an integer or half integer multiple of* $\hbar$
   □ *not quantized, because the length is not an integer or half integer multiple of* $\hbar$

4. The stationary wave function of an electron $\psi(\mathbf{r})$ can never have a break, angle or cusp

   Solution:
   □ *true, because it must always be a differentiable function*
   □ *false because it must always be a differentiable function*
   ✓ *false, because it may have breaks, where there is a potential jump*
   □ *true, because othervise it can't be normalized*

5. The energy levels of an electron in a cubic potential box

   Solution:
   □ *can never be degenerate and are equidistant*
   □ *are degenerate and equidistant*
   ✓ *are degenerate and proportional to the square of integer numbers*
   □ *are degenerate and proportional to the cube of integer numbers*

6. In silver the outermost occupied shell is 5s which has a single, unpaired electron on it. If a beam of neutral silver atoms is shot through an inhomogeneous magnetic field we see N spots on a screen after the magnetic field, where

   Solution:

   ✓ *N=2, because the total angular momentum is the spin momentum only*
   ☐ *N=1 and the spot is in the middle of the screen, because the angular momentum on shell 5s is zero*
   ☐ *N=3, because the total angular momentum on shell 5s is* $\hbar$
   ☐ *N=5, because the angular momentum on shell 5s is* $\hbar + 1/2\,\hbar$

7. Helium has two electrons and, as a consequence of the Pauli principle, these electrons form

   Solution:

   ☐ *a singlet state, because the wave function must be antsymmetric*
   ☐ *a triplet state, because the wave function must be antisymmetric*
   ☐ *both a triplet and a singlet state, because the wave function must be antisymmetric*
   ✓ *both a triplet and a singlet state, because only the product of the wave function and the spin state must be antisymmetric*

8. Hybridized states in molecules

   Solution:

   ☐ *have a well defined angular momentum*
   ☐ *only* sp^3 *states have a well defined momentum*
   ✓ *doesn't have well defined momenta, because momenta in s and p states differ*
   ☐ *only* sp^3 *states in* σ *bonds have well defined momentq*

9. The kinetic energy of a rotating diatomic molecule

   Solution:

   ☐ *is not quantized, because the molecule rotates freely*
   ☐ *is not quantized, only the angular momentum is quantized*
   ✓ *quantized with an energy quantum much smaller than the thermal energy at 300 K*
   ☐ *quantized with an energy quantum much larger than the thermal energy at 300 K*

10. In statistical equilibrium of a system

   Solution:

   ☐ *all macrostates have the same probability*
   ☐ *all macrostates have the same number of microstates*
   ✓ *all microstates have the same probability*
   ☐ *all microstates have the same number of microstates*

2 Problems

Useful constants: elementary charge: $e = 1.6022 \cdot 10^{-19}\ C$,
electron mass: $m_e = 9.1094 \cdot 10^{-31} kg$,
Stefan-Boltzman constant: $\sigma = 5.6704 \cdot 10^{-8}\ W m^{-2}\ K^{-4}$,

11. An electron gun emits electrons whose wavelength is such that it can excite a linear harmonic oscillator of frequency $\nu = 2.20 \cdot 10^{18}$ Hz. What is the minimum accelerating voltage of the electron gun? (**10 points**)

Solution:
The energy of the linear harmonic oscillator can only change in quanta of $h\,\nu$ *therefore the electron must have an integer multiple of this energy to be able to excite this oscillator:*

$$\mathcal{E}_e = h\,\nu\,n, \ \ where\ n = 1, 2, 3...$$

The smallest value is $\mathcal{E}_e = h\,\nu$. *The energy of the electron accelerated through a voltage* U *is* $\mathcal{E}_e = e\,U$ *and this is the kinetic energy it will transfer to the oscillator:*

$$e\,U = h\,\nu \qquad \Rightarrow \qquad U = \frac{h\,\nu}{e} = \frac{6.63 \cdot 10^{-34} Js\, 2.20 \cdot 10^{18} Hz}{1.60 \cdot 10^{-19} J/V} = 9.1 \cdot 10^{3}\ V$$

12. The 3^{rd} excited state in a hydrogen atom have energy of $-1.51 eV$ relative to the vacuum level (=energy of the free electron). What is the wavelength of a photon emitted in a transition between level 2 and the ground level? (**15 points**)

Solution:
In a hydrogen atom $\mathcal{E}_n$ *(*$n = 1, 2, 3...$*) is proportional to* $\frac{1}{n^2}$, *therefore the ground state* $\mathcal{E}_1$ *corresponds to* $n = 1$, *so* $\mathcal{E}_n = \frac{\mathcal{E}_1}{n^2}$. *therefore*

$$\mathcal{E}_3 = \frac{\mathcal{E}_1}{3^2} = \frac{\mathcal{E}_1}{9} \qquad \Rightarrow \qquad CE_1 = 9 \cdot \mathcal{E}_3 = -13.4 eV$$

The frequency is

$$\nu = \frac{\Delta\mathcal{E}}{h} = \frac{1}{h}\mathcal{E}_1 \left(\frac{1}{2^2} - 1\right) = -\frac{3\,\mathcal{E}_1}{4\,h} = 2.43 \cdot 10^{15}\ Hz$$

The wavelength is

$$\lambda = \frac{c}{\nu} = -\frac{4\,h\,c}{3\,\mathcal{E}_1} = 1.23 \cdot 10^{-7}\ m = 123\,nm$$

13. An electron is moving freely along the x axis in a potential which is zero for $x < 0$ values and infinitely large for $x \geq 0$. a) What is/are the boundary condition(s)? b) What is the wave function of the electron for $x < 0$? And for $x \geq 0$? c) Is this wave function normalizable? Why? (**20 points**)

Solution:
a) boundary condition is (The potential has an infinite jump at 0, so just one condition)
$\psi_-(0) = \psi_+(0)$
b) The solutions of the Schrödinger equation for the two regions are:

$$\begin{aligned} &\psi_-(x) = A\,e^{i\,k\,x} + B\,e^{-i\,k\,x} && x < 0 \\ &\psi_+(x) = 0 && x \geq 0 \end{aligned}$$

Using the boundary condition: $A + B = 0$, *so the wave function is*

$$\begin{aligned} &\psi_-(x) = A\,\left(e^{i\,k\,x} - e^{-i\,k\,x}\right) = 2\,i\,A\,\sin(k\,x) && x < 0 \\ &\psi_+(x) = 0 && x \geq 0 \end{aligned}$$

c) The wave function is not normalizable (therefore not physical), because the integral of $|sin^x|$ *over an infinite range isn't finite.*

14. In a 3D cubic potential box with sides $L = 4\,\mu m$ the wavelength of the photons emitted in the transition from level N is $8.79\,m$.
a) What is N?
b) What is the degeneracy of level N? How would the wavelength change if the length of the side of the box was tripled?
Express all numbers in scientific notation (i.e. as $X.YZ \cdot 10^R$) and use exactly 2 decimal digits in the calculation! (**25 points**)

Solution:
a) The stationary wave function can be written as $\psi(x,y,z) = C\, sin\frac{\pi\,x}{L} n_x \cdot sin\frac{\pi\,y}{L} n_y \cdot sin\frac{\pi\,z}{L} n_z$ *so it is separable, therefore in 3D the energy will be a sum of 3 terms. In 1D*

$$L = n\frac{\lambda}{2} \Rightarrow \lambda = 2\frac{L}{n} \Rightarrow p = \frac{h}{\lambda} = \frac{h}{2\,L}\,n \quad \textit{where } n = 1, 2, 3...$$
$$\mathcal{E}_n = \frac{p^2}{2\,m} = \frac{h^2}{8\,m\,L^2}\,n^2$$

Therefore in 3D

$$\mathcal{E}_n = \frac{h^2}{8\,m\,L^2}\,\left(n_x^2 + n_y^2 + n_z^2\right)$$

For a cubic box in 3 dimensions all but the ground level is degenerate. In the ground state ($\mathcal{E}_1$) *all of* n_x, n_y, n_z *must be 1. Write this as* $\mathcal{E}_o \cdot 3$

$$\mathcal{E}_o = \frac{(6.63 \cdot 10^{-34})^2}{8 \cdot 9 \cdot 10^{-31}\,(4.00 \cdot 10^{-6})^2} = 3.77 \cdot 10^{-27}\,J$$
$$\mathcal{E}_1 = 3 \cdot \mathcal{E}_o = 1.13 \cdot 10^{-26}\,J$$

The given transition corresponds to an energy difference of $\mathcal{E}_N - \mathcal{E}_1$ *which can be expressed two ways*

$$\mathcal{E}_N - \mathcal{E}_1 = \mathcal{E}_o\,\left(n_x^2 + n_y^2 + n_z^2 - 3\right) = 3.77 \cdot 10^{-27}\left(n_x^2 + n_y^2 + n_z^2 - 3\right)$$
$$\mathcal{E}_N - \mathcal{E}_1 = h\,\frac{c}{\lambda} = 6.63 \cdot 10^{-34}\,Js\frac{3.00 \cdot 10^8\,m/s}{8.79\,m} = 2.26 \cdot 10^{-26}\,J$$

Let's calculate this using the first of these and compare the result with the second for the first few energy values!

$$\mathcal{E}2 - \mathcal{E}1 = \mathcal{E}_o \cdot (1^2 + 1^2 + 2^2 - 3) = 3 \cdot \mathcal{E}_o = 1.13 \cdot 10^{-26}$$
$$\mathcal{E}3 - \mathcal{E}1 = \mathcal{E}_o \cdot (1^2 + 2^2 + 2^2 - 3) = 6 \cdot \mathcal{E}_o = 2.26 \cdot 10^{-26}$$

So $\underline{\underline{N = 3}}$

b)

Table 1: Possibilities to have the same energy on each level:

level	**combination**	**count**
1	1 1 1	1
2	2 1 1 1 2 1 1 1 2	3
3	3 1 1 1 3 1 1 1 3 2 2 1 2 1 2 1 2 2	6

Degeneracy of level 3 is $\underline{\underline{6}}$

Because $\lambda = 2\,L$ *if* $L' = 3\,L$ *then* $\lambda' = 2 \cdot 3\,L = 2\,\lambda$