Test #1

Quantum Mechanics
March 25, 2024
Solution

You need to have at least 20 points to pass the test

1 Quiz

Select the correct answer from the possibilities. Each correct answer is 2 points, each incorrect one is -1 point, no answer is 0 point, but the total points for one question are never negative.
Total points for this part is 20 points

1. According to (classical) electrodynamics accelerating charges radiate EM waves, because electrons in atoms are orbiting, therefore accelerating, they should loose their kinetic energy and fall into the nucleus, but atoms are stable. Why?

   Solution:
   ☐ *because (classical) electrodynamics is invalid in QM*
   ☐ *because electrons also get energy back from other electrons*
   ✓ *because electrons of stationary orbitals are not accelerating*
   ☐ *because for electrons the Pauli exclusion principle is valid*

2. Value and unit of the Planck constant and 1 electron volt

   Solution:
   ☐ $9.1 \times 10^{-31} J$,$1.6 \times 10^{-19} C$
   ✓ $6.63 \times 10^{-34} Js$, $1.6 \times 10^{-19} J$
   ☐ $9.1 \times 10^{-31} Js$, $1.6 \times 10^{19} As$
   ☐ $1.6 \times 10^{-19} C$, $6.63 \times 10^{-34} Js$

3. Which statement(s) is (are) true?

   Solution:
   ✓ *In classical physics the energy is continuous, while in quantum physics bound states have discreet energy levels.*
   ☐ *Both classical and quantum mechanics the energy always changes in discreet units, but this unit is undetectably small for macroscopic bodies*
   ☐ *It depends on the interaction so no general statement can be given*
   ☐ *In quantum physics and for bound states in classical physics the energy is continuous.*

4. What quantity/quantities is/are called “potential’ in quantum mechanics?

Solution:
✓ *The potential energy and the operator of the potential energy*
□ *The electrical potential of an atom*
□ *The part of the energy that can be used for work*
□ *The internal energy of quantum systems*
□ *The voltage difference between two quantum states*

5. Can a real inter-atomic potential near to the equilibrium position be approximated with a linear harmonic oscillator?

Solution:
□ *Yes, because the potential curve can be approximated by a circle segment.*
✓ *Yes, because the potential curve can be approximated by a parabola.*
□ *It is not an approximation, because that is the potential curve of a linear harmonic oscillator.*
□ *No, because the potential in the molecule is not harmonic.*
□ *No, because the wave function is not a harmonic function.*

6. During tunneling through a narrow potential barrier with a maximum height of V_0

Solution:
□ *when $E > V_0$ the electron is never reflected back*
□ *when $E < V_0$ the electron is always reflected back*
✓ *the probability of passing through decreases exponentially with the width of the barrier*
□ *the probability of passing through increases exponentially with the width of the barrier*
□ *both in classical and in quantum physics, when $E > 1.5 \times V_0$ the electron is never reflected back*

7. In a single measurement of any physical parameter on a micro-particle system the measured result

Solution:
✓ *is always a single eigenvalue of the system, but it cannot be predicted which one before the measurement is performed*
□ *is always a combination of eigenvalues of the system*
□ *is the probability of a given value*
□ *is the expectation value of the operator of the parameter*
□ *is always an eigenvalue of the system, and can be determined which one even before the measurement is performed*

8. The electron orbital in an atom is

Solution:
□ *the region of space inside which the electron resides with a -90% probability*
□ *just another word for the orbit of the electron in the atom*

✓ *another name of the wave function of the electron*
□ *the absolute value of the wave function of the electron*
□ *any of the Bohr-orbits*

9. The blue color of big bodies of water and water ice

   Solution:
   ✓ *comes from the absorption of red light by the vibrating water molecules*
   □ *comes from the scattering of the blue light by the water molecules*
   □ *is caused by Rayleigh-scattering*
   □ *is caused by indirect Reyleigh-scattering*
   □ *is caused by Raman-scattering*

10. For a photon gas you have to

    Solution:
    □ *use Maxwell-Boltzmann statistics*
    □ *use the Pauli principle*
    □ *use Fermi-Dirac statistics*
    ✓ *use Bose-Einstein statistics*
    □ *use the Compton formula*

2 Problems

Useful constants: elementary charge: $e = 1.6022 \cdot 10^{-19}\ C$,
electron mass: $m_e = 9.1094 \cdot 10^{-31} kg$,
Stefan-Boltzman constant: $\sigma = 5.6704 \cdot 10^{-8}\, Wm^{-2}\, K^{-4}$,

11. The new James Webb space telescope works mostly in the near to mid infrared range. For this the telescope must be kept very cold, under 50K. This is achieved to put a five-layer heat shield below the telescope itself, which protects it from the EM radiation of the Sun, the Earth and the Moon. The shield's fully deployed dimensions are $14.162\, m \times 21.197\, m$. Calculate how much thermal energy reaches the shield from the Sun in every second, knowing that the Sun's temperature is $T = 5772\, K$, the Earth-Sun distance is 150.4 million kilometer and the Sun's radius is 696 342 km!

    (*Fun fact: the temperature on the Sun facing side may reach 110 ^oC (383 K), while on the other side it can be as low as -234^oC (39 K), and even so, the mid IR. instrument still must be cooled by a helium refrigerator, or cryocooler system down to 7 K.*) (**10 points**)

    Solution:
    The total power of the radiation from the Sun using the Stephan-Boltzmann law is

    $$P_{tot} = 4\pi \times \sigma \times R_{Sun}^2 \times T_{Sun}^4 (= 3.835 \cdot 10^{26} W)$$

3 points

One m^2 area at $r = 150.4 \cdot 10^9$ m distance from the Sun receives

$$p = \frac{P_{tot}}{4\pi \times r^2} \times 1m^2 = \frac{\cancel{4\pi} \times \sigma \times R_{Sun}^2 \times T_{Sun}^4}{\cancel{4\pi} \times r^2}$$
$$= \left(\frac{R_{Sun}}{r}\right)^2 \times \sigma \times T_{Sun}^4 (= 1.3491 \cdot 10^3\, W)$$

power, *4 points*

so the area $A = 14.162\, m \times 21.197\, m = 3.0019 \cdot 10^2\, m^2$ receives

$$p_A = p \times A = 4.0498 \cdot 10^5 W \quad \textit{power and}$$
$$\mathcal{E}_A = p_A \times 1\, s = \underline{\underline{4.0498 \cdot 10^5 J}} \quad \textit{energy in 1 second}$$

3 points

12. A linear harmonic oscillator with zero point energy $\mathcal{E}_o = 1.1216 \cdot 10^{-17}\, J$ is excited by electrons accelerated through $V = 420\, V$. Are the electrons able to excite the oscillator? (Calculate with at least 4 digits of accuracy)
If your answer is 'Yes' then calculate N, the index of the excited level, otherwise prove excitation with these electrons is not possible!
(Hint: use eV in the calculations.) (**10 points**)

Solution:

Step #1 *7 points*

$\mathcal{E}_N = h\,\nu\,(N + \frac{1}{2})$, $\mathcal{E}_o = \frac{1}{2}\,h\,\nu$, *If there is an integer N for which $V = \Delta\mathcal{E}\,(\equiv \mathcal{E}_N - \mathcal{E}_0)$, then excitation is possible, otherwise it is not.* $\mathcal{E}_N - \mathcal{E}_o = N\,h\,\nu$.

$$h\,\nu = 2\,\mathcal{E}_o = 2\,\frac{1.216 \cdot 10^{-17}\, \cancel{J}}{1.602 \cdot 10^{-19}\, \cancel{J}/eV} = 140\, eV$$

$$\underline{\underline{N}} = \frac{V}{h\,\nu} = \frac{420\,\cancel{eV}}{140.0\,\cancel{eV}} = \underline{\underline{3}}$$

Step #2 *3 points*

Because the value of N is an integer, excitation is possible by these electrons.

Remark: *if the formulas used are correct, but due to an incorrect rounding error in the calculations (when the value of the electric charge used to convert Joules to eV is not given by 4 decimal digits) then value for N is not an integer, which should lead to the conclusion that excitation is not possible, the maximum points are given.*

13. What are the frequency of photons a rotating O_2 molecule can absorb in a transition between its 4th and 5th energy state? ($M_{O_2} = 31.9989$ g/mol, $d_{O-O} = 0.1208\, nm$) (**10 points**)

Solution:

The rotating molecule has energy levels $\mathcal{E}_\ell = \frac{\hbar^2}{2\,I}\,\ell\,(\ell+1)$ so the distance of the $(\ell+1)$th and the ℓth level is (c.f. book Eq. 7.6.5)

$$\Delta\,\mathcal{E}_{\ell\to\ell+1} = \mathcal{E}_{\ell+1} - \mathcal{E}_\ell = \frac{\hbar^2}{I}\,(\ell+1)$$

Calculate I:

$$M_{O_2} = 2\,m_O = \frac{31.9989\,g/mol}{L_A} = \frac{31.9989\,g/\cancel{mol}}{6.02\cdot 10^{23}/\cancel{mol}} = 5.31{\cdot}10^{-23}\,g = 5.31{\cdot}10^{-26}\,kg.$$

3 points

The O_2 molecule is a linear molecule with two equal non-zero moment of inertia (rotational inertia) of

$$I = 2\times m_O\,r^2 = M_{O_2}\,r^2 = M_{O_2}\left(\frac{d_{O-O}}{2}\right)^2 = 5.31\cdot 10^{-26}\,kg\left(\frac{1.208\cdot 10^{-10}\,m}{2}\right)^2$$

$$I = 1.94\cdot 10^{-46}\,kg\,m^2$$

When $\ell = 4$, *then* $\Delta\,\mathcal{E}_{4\to 5} = 5\,\frac{\hbar^2}{I} = 5\times\frac{\left(1.06\cdot 10^{-34}\,J\,s\right)^2}{1.94\cdot 10^{-46}\,kg\,m^2} = \underline{\underline{2.875\cdot 10^{-22}\,J}}$

(units: $\frac{J^2\,s^2}{kg\,m^2} = \frac{J\,kg\,m^2/s^2\,s^2}{kg\,m^2} = J$*)* *(4 points)*

For an absorbed photon: $h\,\nu = \Delta\mathcal{E}_{4\to 5}$ *(3 points)*

$$\nu = \frac{\Delta\mathcal{E}_{4\to 5}}{h} = \underline{\underline{4.3389\cdot 10^{11}}}s^{-1}$$